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Partial coherence in the core–halo picture of Bose–Einstein n-particle correlations

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Abstract. We study the influence of a possible coherent component in the boson source on the two-, three- and n-particle correlation functions in a generalized core–halo-type boson-emitting source. In particular, a simple formula is presented for the strength of the n-particle correlation functions for such systems. Graph rules are obtained to evaluate the correlation functions of arbitrarily high order. The importance of an experimental determination of the 4-th and 5-th order Bose–Einstein correlation function is emphasized.

1 Introduction

Intensity correlations were discovered first in astrophysics by Hanbury Brown and Twiss [1], who invented this method to determine the angular diameter of main sequence stars (HBT effect). In particle physics, intensity correlations of pions were observed by Goldhaber, Goldhaber, Lee and Pais (GGLP effect) [2]. Bose–Einstein correlations are intensity correlations among identical bosons, that are studied mainly with the purpose of reconstructing the space-time picture of particle production. The analysis of higher-order Bose–Einstein correlation functions became a focal point of current research interest.

In particle physics, significant three- or higher-order Bose–Einstein correlations have been extracted from the data samples of the AFS [3], the NA22 [4–6] and the UA1 collaborations [7]. These data were used to test the possible existence of a coherent source in multi-particle physics and to compare the correlation functions with the strength of these correlations as predicted from the quantum-optical (QO) formalism [8–10]. As the precision of the measurements improved, the QO predictions with higher-order correlations were found to be less and less consistent with the data on third- and fourth-order Bose–Einstein correlation functions (BECFs) [11] in $(\pi^+/K^+)+p$ reactions at CERN SPS. Recently, this basic QO formalism was shown to be insufficient to simultaneously describe the high-precision UA1 data on two- and three-particle Bose–Einstein correlations [12].

In high-energy heavy-ion physics, the first experimental determination of the three-particle correlation function has recently been reported by the NA44 collaboration [13–16], indicating that the genuine three-particle correlation is quite suppressed in the S + Pb collisions at CERN SPS. Genuine three-particle correlation means the part of the three-particle correlation that is not due to included combinations of two-particle correlations. This suppression can be expressed as a phase factor, $\cos(\phi)$, of the three-particle correlation function, in case of totally incoherent particle production. In that case the phase factor is related to an asymmetry of the particle source, that cannot be extracted from two-particle correlations. Theoretical estimates of this asymmetry effect on the phase factor show very small departures from $\cos(\phi) \approx 1$ [17–19]. The large departure from $\cos(\phi) = 1$ found by the NA44 collaboration, $\cos(\phi) = 0.2 \pm 0.2$ [16], ought to be due to some other mechanism. We will discuss the possibility of a partially coherent source in this Letter. The possible existence of such an extra phase in the third- and higher-order correlation functions was noted already e.g. in papers by the NA22 collaboration [11], but no experimental evidence has been put forward for a $\cos(\phi) \neq 1$ value in particle physics.

Theoretically Cramer and Kadija predicted up to order 6 the strength of Bose–Einstein correlations for sources with partially coherent and incoherent components that also included a possible contamination by mis-identified, non-interfering particles [20]. Their formulae were obtained in the quantum-optical formalism. Recently, Suzuki and collaborators calculated higher-order exclusive Bose–Einstein correlations from the generating-functional approach to the quantum-optical formalism [21] for the case that the source has M incoherent and one coherent component.

Recently, multi-particle symmetrizations up to arbitrarily high order were evaluated exactly by Zhang [22] for the special case of the pion-laser model proposed by Pratt in [23]. Surprisingly, the structure of the n-particle inclusive correlation functions in terms of the Fourier-

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transformed inclusive emission function was found to be the same as the structure of the n-particle exclusive correlation functions in terms of the single-particle exclusive emission function [22]. However, this result is valid only in the case when Bose–Einstein condensation, and hence the development of partial coherence, has not yet been reached [24].

A simple recurrence relation was obtained for the strength of the higher-order correlation functions of core–halo-type systems [25]. Such systems are boson-emitting sources where some particles come from the incoherent center of the particle emission, which is assumed to be resolvable by the Bose–Einstein microscope. The rest of the particles is assumed to come from the halo region, which corresponds to large length scales not resolvable by intensity interferometry [26,27]. In [25], a prediction was made for the strength of third-order and arbitrary-order BECF assuming that the core has no coherent component.

The purpose of the present Letter is to investigate the effect of a partially coherent component in the core of particle emission. We present a generalization of the earlier recurrence relations in [25]; the new expressions also yield an easy way to calculate formulae for the strength of the n-th order correlation function with a partially coherent and a halo component, and we apply these expressions to the NA44 data on $\mathrm{S}+\mathrm{Pb}$ collisions.

2 Basic definitions

The central assumption of the core–halo model is that the reduction of the intercept parameter of the n-particle BECFs be due to the presence of the long-lived resonances [25]. This assumption was motivated by the success of fully incoherent event generators like RQMD or VENUS in the description of two-particle BECFs.

The emission function of the whole source can be written as a sum of a contribution from the core and from the halo, where the halo stands for the decay products of the (non-resolvable) long-lived resonances. The core is indexed with 'c', the halo by 'h'. We have

$$S(x,k)=S_{\mathrm{c}}(x,k)+S_{\mathrm{h}}(x,k). \tag{1}$$

In earlier studies of the core–halo model it was assumed that $S_{\mathrm{c}}(x,k)$ describes a fully incoherent (thermal) source. Now we assume that some fraction of the core emits bosons in a coherent manner, e.g., due to the emerging formation of pion lasers, or Bose–Einstein condensates of pions, or production of disoriented chiral condensates, etc., so we define

$$S_{\mathrm{c}}(x,\boldsymbol{k})=S_{\mathrm{c}}^{\mathrm{p}}(x,\boldsymbol{k})+S_{\mathrm{c}}^{\mathrm{i}}(x,\boldsymbol{k}), \tag{2}$$

where the upper index 'p' stands for the coherent component (p is for partial), and the upper index 'i' stands for the incoherent component of the core.

The invariant spectrum is given by

$$N(\boldsymbol{k})=\int \mathrm{d}^4x S(x,\boldsymbol{k})=N_{\mathrm{c}}(\boldsymbol{k})+N_{\mathrm{h}}(\boldsymbol{k}), \tag{3}$$

and the core contribution is a sum of the coherent and incoherent components:

$$N_{\mathrm{c}}(\boldsymbol{k})=\int \mathrm{d}^4x S_{\mathrm{c}}(x,\boldsymbol{k})=N_{\mathrm{c}}^{\mathrm{p}}(\boldsymbol{k})+N_{\mathrm{c}}^{\mathrm{i}}(\boldsymbol{k}). \tag{4}$$

One can introduce the momentum-dependent core fractions $f_{\mathrm{c}}(\boldsymbol{k})$ and the partially coherent core fractions $p_{\mathrm{c}}(\boldsymbol{k})$ by

$$f_{\mathrm{c}}(\boldsymbol{k})=N_{\mathrm{c}}(\boldsymbol{k})/N(\boldsymbol{k}), \tag{5}$$

$$p_{\mathrm{c}}(\boldsymbol{k})=N_{\mathrm{c}}^{\mathrm{p}}(\boldsymbol{k})/N_{\mathrm{c}}(\boldsymbol{k}). \tag{6}$$

The halo and the incoherent fractions $f_{\mathrm{h}}, f_{\mathrm{i}}$ are

$$f_{\mathrm{h}}(\boldsymbol{k})=N_{\mathrm{h}}(\boldsymbol{k})/N(\boldsymbol{k})=1-f_{\mathrm{c}}(\boldsymbol{k}), \tag{7}$$

$$f_{\mathrm{i}}(\boldsymbol{k})=N_{\mathrm{c}}^{\mathrm{i}}(\boldsymbol{k})/N_{\mathrm{c}}(\boldsymbol{k})=1-p_{\mathrm{c}}(\boldsymbol{k}). \tag{8}$$

Note that our definition of the momentum-dependent partially coherent core fraction, $p_{\mathrm{c}}(\boldsymbol{k})$, should be clearly distinguished from the chaoticity p of Weiner [28], defined as $p=\langle n_{\mathrm{chao}}\rangle/\langle n_{\mathrm{tot}}\rangle$, the ratio of the mean number of particles from the chaotic source to the mean total multiplicity. If we neglect the momentum dependence of $f_{\mathrm{c}}(\boldsymbol{k})$ and $p_{\mathrm{c}}(\boldsymbol{k})$, the core fraction and the partially coherent core fraction, formally one obtains $p=1-p_{\mathrm{c}}f_{\mathrm{c}}$. However, we distinguish the resolvable intercept λ_* from the exact intercept λ_{xct}, in contrast to [28]. For example, in the case of two-particle correlations of core-halo type systems, $\lambda_{*,2}=f_{\mathrm{c}}^2[(1-p_{\mathrm{c}})^2+2p_{\mathrm{c}}(1-p_{\mathrm{c}})]$, while $\lambda_{xct,2}=\lambda_{*,2}+(1-f_{\mathrm{c}})^2+2f_{\mathrm{c}}(1-f_{\mathrm{c}})$. In the case of the quantum optical formalism without long lived resonances $\lambda_2^{\mathrm{QO}}=2p(1-p)+p^2=1-(1-p)^2$.

3 The strength of the n-particle correlations $\lambda_{*,n}$

We define the n-particle correlation function by

$$\begin{aligned}C_n(1,2,\ldots,n)&=C_n(\boldsymbol{k}_1,\boldsymbol{k}_2,\ldots,\boldsymbol{k}_n)\\&=\frac{N_n(\boldsymbol{k}_1,\boldsymbol{k}_2,\ldots,\boldsymbol{k}_n)}{N_1(\boldsymbol{k}_1)N_1(\boldsymbol{k}_2)\ldots N_1(\boldsymbol{k}_n)}, &(9)\\&=\frac{N_n(1,2,\ldots,n)}{N_1(1)N_1(2)\ldots N_1(n)}, &(10)\end{aligned}$$

where a symbolic notation for $\boldsymbol{k}_i$ is introduced; only the index of $\boldsymbol{k}$ is written out in the argument. From now on, we shall consistently apply this notation for the arguments of the various functions of the momenta, i.e., $f(\boldsymbol{k}_i,\boldsymbol{k}_j,\ldots,\boldsymbol{k}_m)$ is symbolically denoted by $f(i,j,\ldots,m)$.

We find that the intercept of the n-particle correlation function (extrapolated from finite relative momenta to zero relative momentum) is given by the following formula:

$$\begin{aligned}C_n(k_i=k_j,\forall i,j)&=1+\lambda_{*,n} &(11)\\&=1+\sum_{j=2}^{n}\binom{n}{j}\alpha_j f_{\mathrm{c}}^j\left[(1-p_{\mathrm{c}})^j+jp_{\mathrm{c}}(1-p_{\mathrm{c}})^{j-1}\right],\end{aligned}$$

where α_j counts the number of fully mixing permutations of j elements. This can be calculated from a simple recurrence, as obtained in [25].

Note that the equations of [25, 26] were given for a fully incoherent core, and they are modified above for an additional coherent component in a straightforward manner. In general, terms proportional to f_c^j in the incoherent case shall pick up an additional factor $[(1-p_c)^j + jp_c(1-p_c)^{j-1}]$ in case the core has a coherent component. This extra factor means that either each of the j particles is from the incoherent part of the core, or one of them can come from the coherent, while the remaining $j-1$ particles must be from the incoherent part. If two or more particles come from the coherent component of the core, the contribution to the intensity correlations vanishes as the intensity correlator for two coherent particles is zero.

Let us indicate the number of permutations that completely mix exactly j non-identical elements by α_j. There are exactly $\binom{n}{j}$ different ways to choose j different elements from among n different elements. Since all the $n!$ permutations can be written as a sum over the fully mixing permutations, the counting rule yields a recurrence relation for α_j [25]:

$$\alpha_n = n! - 1 - \sum_{j=1}^{n-1} \binom{n}{j} \alpha_j. \tag{12}$$

The first few values of α_j are given by

$$\alpha_1 = 0, \tag{13}$$
$$\alpha_2 = 1, \tag{14}$$
$$\alpha_3 = 2, \tag{15}$$
$$\alpha_4 = 9, \tag{16}$$
$$\alpha_5 = 44, \tag{17}$$
$$\alpha_6 = 265. \tag{18}$$

We have the following explicit expressions for the first few intercept parameters:

$$\lambda_{*,2} = f_c^2[(1-p_c)^2 + 2p_c(1-p_c)], \tag{19}$$
$$\lambda_{*,3} = 3f_c^2[(1-p_c)^2 + 2p_c(1-p_c)] + 2f_c^3[(1-p_c)^3 + 3p_c(1-p_c)^2], \tag{20}$$
$$\lambda_{*,4} = 6f_c^2[(1-p_c)^2 + 2p_c(1-p_c)] + 8f_c^3[(1-p_c)^3 + 3p_c(1-p_c)^2] + 9f_c^4[(1-p_c)^4 + 4p_c(1-p_c)^3], \tag{21}$$
$$\lambda_{*,5} = 10f_c^2[(1-p_c)^2 + 2p_c(1-p_c)] + 20f_c^3[(1-p_c)^3 + 3p_c(1-p_c)^2] + 45f_c^4[(1-p_c)^4 + 4p_c(1-p_c)^3] + 44f_c^5[(1-p_c)^5 + 5p_c(1-p_c)^4]. \tag{22}$$

In the above equations, the effective intercept parameters, the core fraction and the partially coherent fraction are evaluated at a mean momentum $\boldsymbol{K}$, $\lambda_{*,n} = \lambda_{*,n}(\boldsymbol{K})$, $f_c = f_c(\boldsymbol{K})$ and $p_c = p_c(\boldsymbol{K})$.

4 The n-body correlation function

Let us give the closed form for the full correlation function for arbitrarily high orders of n, generalizing the results of [25] for an additional partially coherent component in the source.

Let $\rho^{(n)}$ stand for those permutations of $(1, \ldots, n)$ that mix *all* the numbers from 1 to n and let us indicate by ρ_i the value which is replaced by i in a given permutation belonging to the set of permutations $\rho^{(n)}$. (A superscript is the index to a set of permutations, a subscript stands for a given value.) Then we have $\rho_i \neq i$ for all values of $i = 1, \ldots, n$.

If the partially coherent component is vanishing, the general expression for the n-particle inclusive correlation function $C_n(1, \ldots, n)$ was given in [25] as

$$C_n(1, \ldots, n) = 1 + \sum_{j=2}^{n} \sideset{}{'}\sum_{i_1, \ldots, i_j = 1}^{n} \sum_{\rho^{(j)}} \prod_{k=1}^{j} f_c(i_k) \tilde{s}_c(i_k, i_{\rho_k}). \tag{23}$$

Here $\sum'$ indicates that the summation should be taken over those sets of values of the indices that do not contain any value more than once, and the normalized Fourier-transformed emission function of the core is

$$\tilde{s}_c(i,j) = \frac{\tilde{S}_c(i,j)}{\tilde{S}_c(i,i)}; \tag{24}$$

$$\tilde{s}_c(i,j) = \tilde{s}_c^*(j,i) \frac{\tilde{S}_c(j,j)}{\tilde{S}_c(i,i)} \neq \tilde{s}_c^*(j,i). \tag{25}$$

In the above equations, the tilde denotes Fourier transformation over the relative momenta,

$$\tilde{S}_c(l,m) = \int d^4x \exp[i(k_l - k_m) \cdot x] S_c\left(x, \frac{k_l + k_m}{2}\right), \tag{26}$$

and similar expressions hold for the coherent and the incoherent components of the core. [1]

The expression in (23) is valid not only for the case when exactly n bosons are in the system and full symmetrization is performed. $C_n(1, 2, \ldots, n)$ stands for the n-particle exclusive correlation function and $\tilde{S}_c(i,j)$ stands for the Fourier-transformed core-emission function without modifications due to multi-particle symmetrization [22, 24]. In addition, (23) is also valid when the only source of correlations between the pions is due to Bose–Einstein symmetrization, the number of pions is randomly varying from event to event, and $C_n(1, 2, \ldots, n)$ is interpreted as the n-particle inclusive correlation function [22, 24], and $\tilde{S}_c(i,j)$ includes all higher-order symmetrization effects.

[1] Note that with this definition the normalized Fourier-transformed emission function becomes asymmetric under the exchange of the arguments and complex conjugation, although the relationship $\tilde{S}_c(i,j) = \tilde{S}_c^*(j,i)$ is satisfied.

However, (23) is valid only if the core has no partially coherent component. If a coherent component is present, one can introduce the normalized incoherent and partially coherent core fractions by

$$\tilde{s}_{\mathrm{c}}^{\mathrm{i}}(j,k) = \frac{\tilde{S}_{\mathrm{c}}^{\mathrm{i}}(j,k)}{\tilde{S}_{\mathrm{c}}^{\mathrm{i}}(j,j)}, \tag{27}$$

$$\tilde{s}_{\mathrm{c}}^{\mathrm{p}}(j,k) = \frac{\tilde{S}_{\mathrm{c}}^{\mathrm{p}}(j,k)}{\tilde{S}_{\mathrm{c}}^{\mathrm{p}}(j,j)}, \tag{28}$$

and we obtain

$$\begin{aligned} C_n(1,\ldots,n) = 1 + \sum_{j=2}^{n} \sum_{m_1,\ldots,m_j=1}^{n\;\;\prime} \sum_{\rho^{(j)}} & \\ \times \Bigg\{ \prod_{k=1}^{j} f_{\mathrm{c}}(m_k)[1-p_{\mathrm{c}}(m_k)]\ \tilde{s}_{\mathrm{c}}^{\mathrm{i}}(m_k, m_{\rho_k}) & \\ + \sum_{l=1}^{j} f_{\mathrm{c}}(m_l) p_{\mathrm{c}}(m_l)\ \tilde{s}_{\mathrm{c}}^{\mathrm{p}}(m_l, m_{\rho_l}) & \\ \times \prod_{k=1, k\neq l}^{j} f_{\mathrm{c}}(m_k)[1-p_{\mathrm{c}}(m_k)]\, \tilde{s}_{\mathrm{c}}^{\mathrm{i}}(m_k, m_{\rho_k}) \Bigg\}. & \end{aligned} \tag{29}$$

This expression contains phases in the Fourier-transformed normalized source distributions. Actually, two (momentum-dependent) phases are present: one denoted by $\phi^{\mathrm{i}}(\boldsymbol{k}_m, \boldsymbol{k}_n)$ in the Fourier-transformed normalized *incoherent* core-emission function, $\tilde{s}_{\mathrm{c}}^{\mathrm{i}}(\boldsymbol{k}_m, \boldsymbol{k}_n)$ and another independent phase denoted by $\phi^{c}(\boldsymbol{k}_m, \boldsymbol{k}_n)$ is present in the Fourier-transformed normalized *coherent* core-emission function, $\tilde{s}_{\mathrm{c}}^{\mathrm{p}}(\boldsymbol{k}_m, \boldsymbol{k}_n)$. One can write

$$\tilde{s}_{\mathrm{c}}^{\mathrm{i}}(\boldsymbol{k}_m, \boldsymbol{k}_n) = |\tilde{s}_{\mathrm{c}}^{\mathrm{i}}(\boldsymbol{k}_m, \boldsymbol{k}_n)| \exp[\mathrm{i}\phi^{\mathrm{i}}(\boldsymbol{k}_m, \boldsymbol{k}_n)], \tag{30}$$

$$\tilde{s}_{\mathrm{c}}^{\mathrm{p}}(\boldsymbol{k}_m, \boldsymbol{k}_n) = |\tilde{s}_{\mathrm{c}}^{\mathrm{p}}(\boldsymbol{k}_m, \boldsymbol{k}_n)| \exp[\mathrm{i}\phi^{\mathrm{p}}(\boldsymbol{k}_m, \boldsymbol{k}_n)]. \tag{31}$$

The shape of both the coherent and the incoherent components is arbitrary in these equations, but should correspond to the space-time distribution of the particle production. If the variances of the core are finite, the emission functions are usually parameterized by Gaussians. If the core distributions have power-law-like tails, as in the case of the Lorentzian distribution [29], then the Fourier-transformed emission functions correspond to exponentials or to power-law structures [30]. For completeness, we list these possibilities below:

$$|\tilde{s}_{\mathrm{c}}^{\mathrm{i}}(\boldsymbol{k}_m, \boldsymbol{k}_n)|^2 = \exp(-R_i^2 Q_{mn}^2) \qquad \text{or} \tag{32}$$

$$|\tilde{s}_{\mathrm{c}}^{\mathrm{i}}(\boldsymbol{k}_m, \boldsymbol{k}_n)|^2 = \exp(-R_i Q_{mn}) \qquad \text{or} \tag{33}$$

$$|\tilde{s}_{\mathrm{c}}^{\mathrm{i}}(\boldsymbol{k}_m, \boldsymbol{k}_n)|^2 = a_i (R_i Q_{mn})^{b_i} \qquad \text{etc. } \ldots, \tag{34}$$

$$|\tilde{s}_{\mathrm{c}}^{\mathrm{p}}(\boldsymbol{k}_m, \boldsymbol{k}_n)|^2 = \exp(-R_p^2 Q_{mn}^2) \qquad \text{or} \tag{35}$$

$$|\tilde{s}_{\mathrm{c}}^{\mathrm{p}}(\boldsymbol{k}_m, \boldsymbol{k}_n)|^2 = \exp(-R_p Q_{mn}) \qquad \text{or} \tag{36}$$

$$|\tilde{s}_{\mathrm{c}}^{\mathrm{p}}(\boldsymbol{k}_m, \boldsymbol{k}_n)|^2 = a_p (R_p Q_{mn})^{b_p} \qquad \text{etc. } \ldots \tag{37}$$

In the above equations, the subscripts 'i' and 'p' index the parameters belonging to the incoherent or to the partially coherent components of the core, and Q_{mn} stands for certain experimentally defined relative momentum components determined from $\boldsymbol{k}_m$ and $\boldsymbol{k}_n$.

A straightforward counting yields that in the limiting case when all momenta are equal, the simple formula of (11) follows from the shape of the n-particle Bose–Einstein correlation functions of (29), as $\tilde{s}_{\mathrm{c}}^{\mathrm{i}}(i,i) = \tilde{s}_{\mathrm{c}}^{\mathrm{p}}(i,i) = 1$.

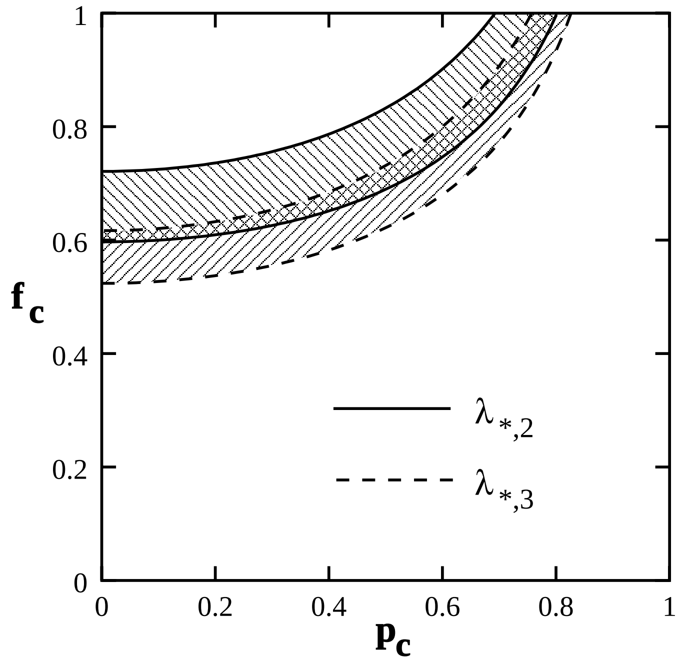

Fig. 1. Allowed regions for possible values of the f_{c} core fraction and the p_{c} partially coherent fraction are evaluated on the 2σ level from the intercept of the second-order and the third-order correlation functions, $\lambda_{*,2}$ and $\lambda_{*,3}$

5 Application to three-particle correlation data

As an application of the above formalism, we attempt to determine the core fraction f_{c} and the partially coherent fraction p_c from the strength of the NA44 two- and three-particle correlation functions, $\lambda_{*,2}$ and $\lambda_{*,3}$, in the CERN SPS S + Pb reactions. The two experimentally determined values are $\lambda_{*,2} = 0.44 \pm 0.04$ and $\lambda_{*,3} = 1.35 \pm 0.12$ (statistical errors only). [2] Figure 1 illustrates the 2σ contour plots in the (f_c, p_c) plane, as determined from the experimental values of $\lambda_{*,2}$ and $\lambda_{*,3}$.

The overlap area in Fig. 1 shows that a large range of (f_c, p_c) is allowed that describe simutaneously the strength of the two- and three-particle correlation functions within two standard deviations. Thus, neither the fully incoherent, nor the partially coherent source picture can be excluded at present.

[2] Coulomb corrections are large in heavy-ion collisions and the value of $\lambda_{*,3}$ was determined with the help of a newly developed Coulomb three-particle wave-function integration method described in [32].

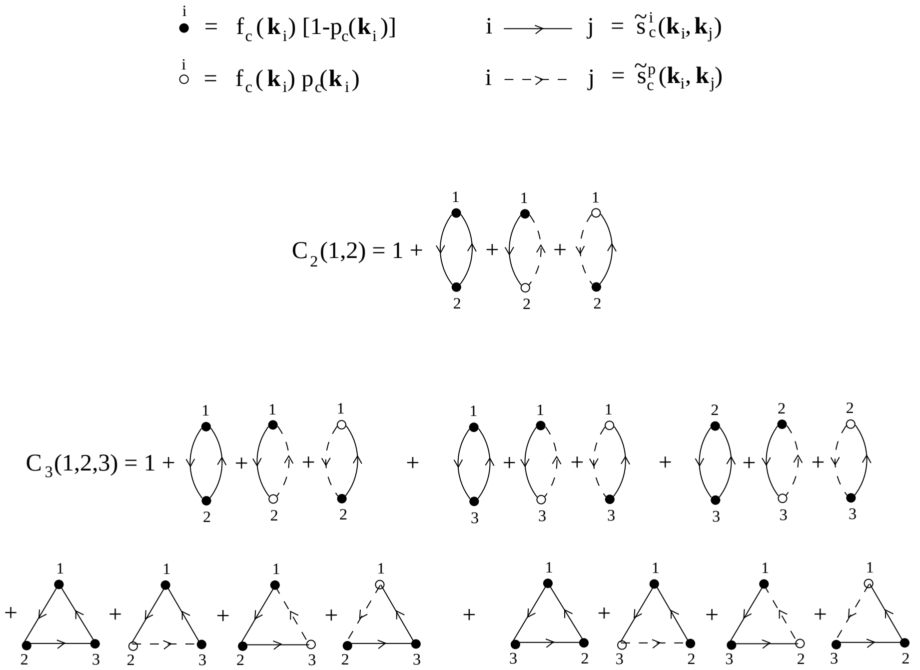

Fig. 2. Graphs determining the second- and the third-order correlation function for partially coherent core–halo sources

Table 1. Evaluation of the strength of the higher-order correlation functions, $\lambda_{*,n}$, for various core fractions and partially coherent fractions allowed by NA44 two- and three-particle correlation data

f_c	p	$\lambda_{*,2}$	$\lambda_{*,3}$	$\lambda_{*,4}$	$\lambda_{*,5}$
0.60	0.00	0.36	1.5	5.1	17.2
0.70	0.50	0.37	1.4	4.3	11.9
1.00	0.75	0.44	1.6	4.3	10.5

Now we can predict the intercept of higher-order correlations to see if they become more sensitive to the presence of a partially coherent source, or not. In Table 1 we have evaluated the the $\lambda_{*,2}, \lambda_{*,3}, \lambda_{*,4}, \lambda_{*,5}$ values for some cases in the overlap region. We find that $\lambda_{*,5}$ is almost a factor of 2 larger for a completely incoherent source than for a partially coherent source with no halo component, although within the experimental errors both cases describe $\lambda_{*,2}$ and $\lambda_{*,3}$. This is in agreement with Cramer and Kadija, who have pointed out that for higher values of n the difference between a partially coherent source and the fully incoherent source will become larger and larger [20].

The results presented here imply that the measurement of higher-order correlations, namely to 5-th order, is necessary to determine the value of the degree of partial coherence of the source in this reaction.

6 Summary and conclusions

In summary, we have found a simple generalization of the core–halo model for the case when the core has a partially coherent component. The strength of the n-particle correlation function can be evaluated for an arbitrary value of n with the help of a simple recurrence formula.

The shape of the n-particle Bose–Einstein correlation function was determined in terms of the Fourier-transformed emission function of the incoherent and the partially coherent component of the source. The graph rules for the calculation of these functions are summarized and illustrated graphically in Appendix A.

We found that the strengths of the second- and the third-order Bose–Einstein correlation functions in the NA44 S + Pb reaction at CERN SPS can be accommodated simultaneously both in a fully incoherent core picture ($p_c = 0$) with a halo fraction of $f_c = 0.6$ as well as in a partially coherent core picture that has no halo component, $p_c = 0.75$ and $f_c = 1$. However, the strength of the fourth- and fifth-order correlation functions is shown to be quite different in the two scenarios.

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Appendix A: Graph rules

A straightforward calculation of the higher-order Bose–Einstein correlations for a partially coherent core–halo type of systems is possible with the help of the set of graph

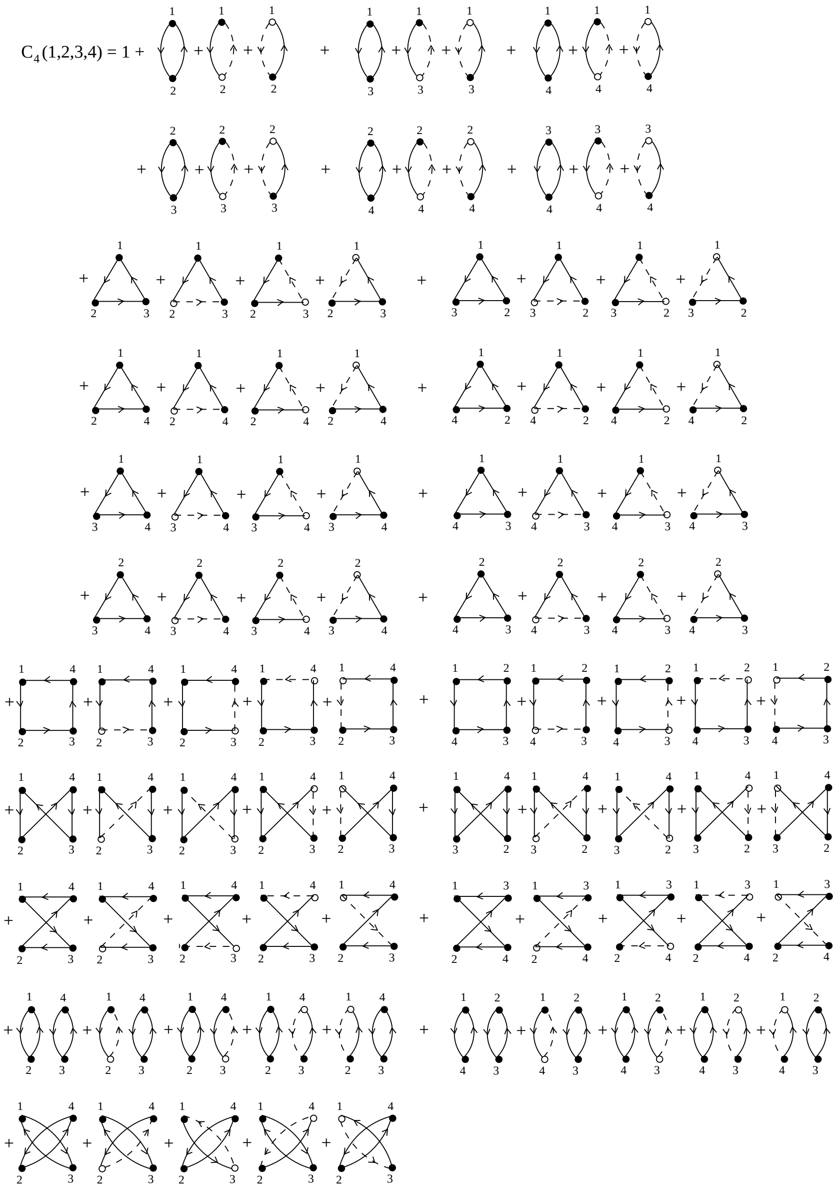

Fig. 3. Graphs determining the fourth-order correlation function for partially coherent core–halo sources

rules that we determine below. Although the graphs we describe are similar to those of [21], the rules are different as we have multiplicative factors for each vertex (they carry one momentum label each) and for each line (that connect two vertices, hence they carry two momentum labels each).

Figures 2 and 3 graphically illustrate the calculation rules of the contributions of the incoherent and coherent core components to the n-particle correlation function $C_n(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n)$ for the cases $n = 2$, 3 and 4, respectively. Circles can be either open or filled. Each circle carries one label (e.g. j) standing for a particle with momentum $\boldsymbol{k}_j$. Filled circles represent the incoherent core component, yielding a factor $f_{\rm c}(j)[1 - p_{\rm c}(j)]$, whereas open circles correspond to the coherent component of the core, a factor $f_{\rm c}(j)p_{\rm c}(j)$, as defined in (6) and (8) and as also shown in Fig. 2.

For the n-particle correlation function, all possible j-tuples of particles have to be found. Such j-tuples can be chosen in $\begin{pmatrix} n \\ j \end{pmatrix}$ different manners. In such a j-tuple, either each circle is filled, or the circle with index k is open and the other $j-1$ circles are filled, which gives $j+1$ different possibilities. All the permutations that fully mix either $j = 2$, or 3, $\ldots$, or n different elements have to be taken into account for each choice of filling the circles. The number of different fully mixing permutations that permute the elements $i_1, \ldots i_j$ is given by α_j and can be determined from the recurrence relation (12).

Lines connecting two circles (or vertices) are denoted, e.g., by (i, j). The lines stand for factors that depend both on $\boldsymbol{k}_i$ and on $\boldsymbol{k}_j$. Full lines represent incoherent–incoherent particle pairs, and correspond to a factor of $\tilde{s}^{\rm i}_{\rm c}(i, j)$. Dashed lines correspond to incoherent–coherent pairs, and carry a factor of $\tilde{s}^{\mathrm{p}}_{\mathrm{c}}(i,j)$. The lines are oriented; they point from circle i to circle j, corresponding to the permutation that replaces element j by element i. Dashed lines must start from an open circle and point to a filled circle.

All possible graphs must be drawn that are in agreement with the above rules. The result corresponds to the fully mixing permutations of all possible j-tuples ($j = 2, \dots n$) chosen in all possible manners from the elements $(1, 2, \dots, n)$.

Each graph adds one term to the correlation function, given by the product of all the factors represented by the circles and lines of the graph. Note that the directions of the arrows matter, as reflected by the inequality in (25). The correlation function $C(1, \dots, n)$ is given by 1 plus the sum of all the graphs.

Finally, we note that for the n-particle *cumulant* correlation function, n circles, representing the n particles, should be connected in all possible manners corresponding only to the fully mixing permutations of the elements $(1, \dots, n)$. Disconnected graphs do not contribute to the cumulant correlation functions, as they correspond to permutations that either do not mix all of the n elements or can be built up from two or more independent permutations of certain subsamples of the elements $(1, 2, \dots, n)$.

References

1. R. Hanbury-Brown, R.Q. Twiss, Phil. Mag. **45**, 663 (1954); R. Hanbury-Brown, R.Q. Twiss, Nature (London) **177**, 27 (1956) and **178**, 1046 (1956)
2. G. Goldhaber, S. Goldhaber, W. Lee, A. Pais, Phys. Rev. **120**, 300 (1960)
3. T. Akesson et al., AFS Coll., Z. Phys. C **36**, 517 (1987)
4. V.V. Aivazyan et al., NA22 Coll., Z. Phys. C **51**, 167 (1991)
5. I.V. Ajinenko et al., NA22 Coll., Z. Phys. C **61**, 567 (1994)
6. N.M. Agababyan et al., NA22 Coll., Phys. Lett. B **322**, 458 (1994)
7. N. Neumeister et al., UA1 Coll., Phys. Lett. B **275**, 186 (1992)
8. R.J. Glauber, Phys. Rev. **131**, 2766 (1963)
9. M. Bijayima, A. Bartle, T. Mizoguchi, N. Suzuki, O. Terazawa, Prog. Theor. Phys. **84**, 931 (1990); **88**, 157 (1992)
10. I.V. Andreev, R.M. Weiner, Phys. Lett. B **253**, 416 (1991); M. Plümer, L.V. Razumov, R.M. Weiner, Phys. Lett. B **286**, 335 (1992)
11. N.M. Agababyan et al., NA22 Coll., Z. Phys. C **68**, 229 (1995)
12. H.C. Eggers, P. Lipa, B. Buschbeck, Phys. Rev. Lett. **79**, 197 (1997)
13. J. Schmidt-Sørensen, NA44 Coll., Nucl. Phys. A **638**, 471 (1998)
14. B. Lörstad, invited talk, Proc. *Correlations and Fluctuations '98*, Mátraháza, Hungary, June 14–21 1998, edited by T. Csörgő, S. Hegyi, R.C. Hwa, G. Jancsó (World Scientific, Singapore 1999)
15. J. Schmidt-Sørensen, in *Proceedings of CRIS '98*, Acicastello, Italy, June 8–12, 1998, edited by S. Costa, S. Albergo, A. Insolia, C. Tove (World Scientific, Singapore 1999)
16. H. Bøggild et al., NA44 Collaboration, Three-pion correlations in S + Pb collisions at CERN SPS, submitted to Phys. Lett. B
17. H. Heiselberg, A.P. Vischer, NBI-96-32, nucl-th/9707036
18. H. Heiselberg, A.P. Vischer, nucl-th/9609023, Phys. Rev. C **55**, 874 (1997)
19. U. Heinz, Q.H. Zhang, nucl-th/9710023, Phys. Rev. C **56**, 426 (1997)
20. J.G. Cramer, K. Kadija, Phys. Rev. C **53**, 908 (1996)
21. N. Suzuki, M. Bijayima, I.V. Andreev, Phys. Rev. C **56**, 2736 (1997)
22. Q.H. Zhang, Phys. Rev. C **58** 18 (1998); Q.H. Zhang, Nucl. Phys. A **634**, 190 (1998)
23. S. Pratt, Phys. Lett. B **301**, 795 (1993)
24. T. Csörgő, J. Zimányi, Phys. Rev. Lett. **80**, 916 (1998); J. Zimányi, T. Csörgő, hep-ph/9705432 - v3; T. Csörgő, J. Zimányi, hep-ph/9811283
25. T. Csörgő, Phys. Lett. B **409**, 11 (1997)
26. T. Csörgő, B. Lörstad, J. Zimányi, Z. Phys. C **71**, 491 (1996)
27. J. Bolz et al., Phys. Rev. D **47**, 3860 (1993); J. Bolz et al., Phys. Lett. B **300**, 404 (1993)
28. R.M. Weiner, Phys. Lett. B **232**, 278 (1989)
29. T. Csörgő, B. Lörstad, Phys. Rev. C **54**, 1390 (1996)
30. A. Bialas, Acta Phys. Pol. B **23**, 561 (1992)
31. S. Nickerson, T. Csörgő, D. Kiang, Phys. Rev. C **57**, 3251 (1998)
32. E.O. Alt, T. Csörgő, B. Lörstad, J. Schmidt-Sørensen, Coulomb corrections to the three-body correlation function in high energy heavy-ion reactions, hep-ph/9812474, submitted to Phys. Lett. B